Physics Letters B 743 (2015) 403–407

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

BICEP2, non-Bunch–Davies and entanglement

Namit Mahajan

Theoretical Physics Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

ARTICLE INFO

Article history:
Received 21 July 2014
Received in revised form 29 January 2015
Accepted 6 March 2015
Available online 9 March 2015
Editor: S. Dodelson

ABSTRACT

BICEP2 result on the tensor to scalar ratio r indicates a blue tilt in the primordial gravitational wave spectrum. This blue tilt and the observed large value $r = 0.2$ are difficult to accommodate within the single field inflationary scenarios under standard conditions. Non-Bunch–Davies vacuum states have been proposed as a possibility. Such vacua are known to lead to pathologies. In this note we point out that it is known that these states ought to be interpreted as excited/squeezed states built over the standard Bunch–Davies vacuum in order to avoid pathological issues. We discuss the associated entanglement properties due to de Sitter horizon, and how such an approach may be more natural in the context of inflation. In particular, we suggest to employ entanglement considerations in de Sitter background to study the nature and intrinsic properties of modified initial states.

In the last one decade or so cosmology has entered the precision era with WMAP [1] and PLANCK [2] dramatically improving upon the COBE observations [3] and providing an accurate set of measurements for other cosmological parameters. Very recently, BICEP2 announced the result on the measurement of the ratio of tensor to scalar spectrum, r [4]. The quoted result for r is $0.2^{+0.07}_{-0.05}$ (with foreground subtraction the central value reads $r = 0.16$). If the BICEP result is confirmed by other experiments, this would constitute a direct signature of primordial gravitational waves. PLANCK Collaboration had provided with an upper limit on r [2], though at different ℓ value than that of BICEP2. There is therefore some tension between the two values.

Inflationary paradigm [5] predicts the power spectra of both the scalar and the gravitational amplitudes (see for example [6] for a quick introduction to the basics and relevant expressions). However, a large value of r as measured by the BICEP2 Collaboration is difficult to achieve in the usual single field slow roll models. Typically, these models predict that the scalar spectral index $n_s - 1 = d\ln P_S/d\ln k \sim -2\epsilon$ while the tensor spectral index $n_T = d\ln P_T/d\ln k = -2\epsilon$. There is an additional 'consistency relation', $r = 16\epsilon = -8n_T$, which ties the two in a simple but strong fashion. The main difficulty, apart from satisfying the large value of r, is the fact that an inflationary model should predict a positive (implying a blue tilt for the gravitational wave spectrum as opposed to the observed red tilt for the scalar power spectrum) and large n_T which is hard to achieve in simple models. Attempts have been made to reconcile the BICEP2 result and the PLANCK upper limit by relaxing some of the conditions/constraints while fitting to the data sets [7]. Another interesting possibility is to consider departure from the usual Bunch–Davies vacuum to get enhanced spectra [8] (see also [9] for some early work).[1] In this note we consider non-Bunch–Davies states and discuss their properties and other related issues. It may be worthwhile to mention at the very outset that whether or not the BICEP2 result continues to hold, which is taken as one of the motivations for looking at non-Bunch–Davies or modified initial states, systematic study of the properties of non-standard states and possible avenues to investigate their nature is still worthy of investigation in it own right. We proceed with this spirit.

We begin by first recalling some of the properties and pathological implications of non-Bunch–Davies vacuum states (for the issues about quantum field theory in de Sitter space that are relevant for the present discussion see [10]). For simplicity consider a scalar filed φ in the de Sitter space. The arguments can be straightforwardly carried over to the graviton. It is known that there is no unique vacuum state in a general curved space–time. Given a quantum field, one expands it in terms of a complete set as

$$\varphi = \sum_n [a_n \varphi_n^{(+)} + a_n^\dagger \varphi_n^{(-)}] \tag{1}$$

[1] Note however that A. Arvind et al. in [8] show that under some reasonable assumptions, the excited/modified states do not alleviate the super-Planckian excursion issue which inflicts the standard scenarios.

E-mail address: nmahajan@prl.res.in.

http://dx.doi.org/10.1016/j.physletb.2015.03.010

where a_n and $\varphi_n^{(+)}$ are the annihilation operators and positive energy mode functions. Since there is no unique vacuum state, one could expand the same field in terms of a different set of mode functions:

$$\varphi = \sum_n [\tilde{a}_n \tilde{\varphi}_n^{(+)} + \tilde{a}_n^\dagger \tilde{\varphi}_n^{(-)}] \tag{2}$$

The two sets are related to each other via the Bogoliubov transformations:

$$\tilde{\varphi}_n^{(+)} = \varphi_n^{(+)} \cosh\alpha_n + \varphi_n^{(-)} e^{i\beta_n} \sinh\alpha_n \tag{3}$$

or in terms of the creation and annihilation operators[2]

$$\tilde{a}_n = a_n \cosh\alpha_n - a_n^\dagger e^{-i\beta_n} \sinh\alpha_n \tag{4}$$

This again displays the fact that the positive frequency modes do not remain positive frequency modes unless α_n are chosen such that the second term above vanishes. In the de Sitter space–time, the common choice is the so called Euclidean or the Bunch–Davies vacuum (sometimes also referred to as thermal vacuum) [11]. In this case, one solves the equation of motion (in conformal time, η) and then demanding that the solution has the correct Minkowski form for high frequencies as $\eta \to -\infty$, one picks out one of the two independent solutions. The end result is (for $\eta \to -\infty$)

$$\varphi_k^{(+)} = H^{(2)}(k\eta) \to \left(\frac{2}{\pi k\eta}\right) e^{-ik\eta} \tag{5}$$

and $\varphi_k^{(-)} = (\varphi_k^{(+)})^* = H^{(1)}(k\eta)$ where $H^{(1,2)}$ are the Hankel functions. We therefore have two vacua: $a_n|E\rangle = 0$ and $\tilde{a}_n|\alpha\rangle = 0$. In the notation we follow, $|E\rangle$ denotes the standard Bunch–Davies or Euclidean vacuum and $|\alpha\rangle$ denotes the other vacuum choice, called α-vacua [12]. $\alpha = 0$ leads to the Euclidean vacuum. It is known that Eq. (1) respects the de Sitter invariance of the background space–time, while for Eq. (2) this happens only if $\alpha_n = \alpha$ (i.e. n-independent). We therefore choose to work with this choice. Further, $\beta_n \neq 0$ in Eq. (3) implies time reversal non-invariance (and also CPT). We thus set β_n to zero. The Euclidean vacuum is easily obtained from the Euclidean de Sitter i.e. sphere via analytic continuation and results in the well known two point functions

$$G_E(x, x') = \langle E|\varphi(x)\varphi(x')|E\rangle \propto {}_2F_1\left(h_+, h_-, \frac{d}{2}, \frac{1+Z}{2}\right) \tag{6}$$

where ${}_2F_1$ is the hypergeometric function, d is the space–time dimensionality, $h_\pm = (d-1)/2 \pm i\nu$ with $\nu = \sqrt{m^2 - \left(\frac{d-1}{2}\right)^2}$ and Z is the invariant distance between two points. $Z(x, x')$ is greater (less) than unity for time-like (space-like) intervals while its is unity for light-like separations. In the above equation, we have considered the two point function (Wightman function) but we shall interchangeably talk about the Hadamard function $G^{(1)}$ which is nothing but the vacuum expectation of the anti-commutator. This function captures the state dependence. One can now evaluate Hadamard function $G_\alpha^{(1)}$ in terms of the Euclidean modes

$$G_\alpha^{(1)}(x, x') = \cosh 2\alpha\, G_E^{(1)}(Z) + \sinh 2\alpha\, G_E^{(1)}(-Z) \tag{7}$$

which shows that the α vacuum function not only depends on Z but also on the antipodal point $-Z$. This feature shows up in other two point functions like the Feynman propagator as well and it is this which is at the heart of pathologies. Specifically, the two point functions have singularities at the antipodal points. Such points are separated by a horizon.[3] Moreover, α vacuum two point functions have non-Minkowski analytic structure in the coincident limit, coming via the contributions due to negative frequency modes. These observations already raise serious doubts about interpreting α vacua as sensible vacuum states. The difficulty with this becomes more serious in an interacting field theory.

Before proceeding further, let us introduce an equivalent way of representing the α modes:

$$\tilde{\varphi}_n^{(+)} = N_{\alpha'}[\varphi_n^{(+)} + \varphi_n^{(-)} e^{\alpha'}] \tag{8}$$

where $N_{\alpha'} = 1/\sqrt{1 - e^{\alpha' + \alpha'^*}}$ and α' a complex parameter with $Re(\alpha') < 0$ and the Euclidean vacuum is obtained for $Re(\alpha') \to -\infty$. Again, a complex α' will lead to troubles with CPT and thus we choose it real. The two representations are related to each other via the variable transformation $e^{\alpha'} = \tanh\alpha$.

As already discussed above, the α vacuum correlation functions have peculiar properties even in the free field theory limit and one would not want to consider them any further. Before doing so, it is worthwhile to have some more check. Consider an Unruh detector moving along a time-like geodesic and with a linear coupling to the field [14]. Denoting by $\mathcal{D}$ the operator which acts on the states of the detector ($|\mathcal{E}_i\rangle$), the detector-field coupling is of the form $\int dt\, \varphi(t)\mathcal{D}(t)$. The transition rate between two states of the detector can be evaluated to be

$$\begin{aligned}\mathcal{R}_\alpha^{ij} &= \mathcal{R}_\alpha^{\mathcal{E}_i \to \mathcal{E}_j} \\ &= |\langle\mathcal{E}_j|\mathcal{D}|\mathcal{E}_i\rangle|^2 \int dt e^{-i(\mathcal{E}_j - \mathcal{E}_i)t} G_\alpha(x(t), x(0))\end{aligned} \tag{9}$$

Using the explicit form (and properties) of G_α it is straightforward to convince oneself that only for the choice of the Euclidean vacuum i.e. $\alpha = 0$ (or equivalently $\alpha' = -\infty$), one has

$$\frac{\mathcal{R}_{\alpha=0}^{ij}}{\mathcal{R}_{\alpha=0}^{ji}} = e^{-2\pi(\mathcal{E}_j - \mathcal{E}_i)/H} \tag{10}$$

which is the expected thermal behaviour and gives the associated temperature to be $T = H/(2\pi)$ in conformity with the Gibbons–Hawking temperature of the de Sitter space. For any other choice of α one therefore has a general result that the detailed balance may not work. This observation again singles out the Euclidean or the Bunch–Davies vacuum as a preferred and unique choice. Although the problems associated with the antipodal point and also the response of Unruh detector in de Sitter have been discussed before (see for example M. Spradlin et al. in [10]), we felt it worthwhile to summarise them.

Let us next ask if there is a way to interpret the α vacua. The Bogoliubov transformation Eq. (3) can be formally thought of as achieved via the action of a unitary operator (this is all familiar from quantum optics where squeezed states are constructed by the action of such unitary operators [15]):

$$S(\xi) = \exp[\sum_n \frac{1}{2}(\xi a_n^{\dagger 2} - \xi^* a_n^2)] \tag{11}$$

such that $S(\xi) a_n S(\xi)^\dagger = \tilde{a}_n$. In the α' representation, $\xi = \frac{1}{4}\ln[\tanh(-\alpha'/2)]$. Now, instead of trying to interpret the α vacua

[2] A general Bogoliubov transformation takes the form: $\tilde{a}_n = A_{nn'}^* a_{n'} - B_{nn'}^* a_{n'}$ with the condition $|A|^2 - |B|^2 = 1$.

[3] Recall that the de Sitter space has a horizon and also carries Gibbons–Hawking temperature $T_{dS} = H/(2\pi)$ [13], where the Ricci scalar $R = 12H^2$.

as genuine vacuum states, one could take the view point that these are excited states built over the Euclidean vacuum.[4] In the context of inflation, squeezed states have been considered before, see [16]. Given these excited/squeezed states, it is then straightforward to evaluate various correlators and one hopes that various pathological issues disappear. However, once again we are confronted with the situation that the expectation value of the two point (and higher point) functions of the quantum field in these states $|\alpha\rangle$ depends on the antipodal point leading to non-local behaviour. This is possible if the states of the system are entangled leading to EPR [17] like effects.

We now ask the question if such a thing is plausible or does one have to simply assume it if these states are to be put to any use. Since the de Sitter space, like a black hole, has a horizon, the space naturally gets divided into two regions. This therefore implies that given a quantum field, there will be non-trivial correlations between the modes inside and outside the horizon. The modes outside (inside) the horizon are not accessible to an observer within (outside) the horizon and therefore the natural approach would be to trace the modes outside (inside) the horizon and use the reduced density matrix thus obtained. With an appropriate choice of α (or α'), the desired squeezed state $|\alpha^{(')}\rangle$ can therefore be constructed. This state is clearly not a pure state but a mixed state and therefore we can expect non-local EPR like effects. In the context of inflation, one can think of the scale at which a given mode exits the horizon: $k_* = aH$. The Fourier modes of the quantum field now naturally get divided in two sets — modes with $k < k_*$ and $k > k_*$. In an interacting field theory (interactions can be in the matter sector itself like φ^4 terms or can eventually arise due to gravity), this will lead to momentum space entanglement between these two set of modes. The end result, qualitatively, is a mixed state after tracing out one set of the modes, but now in the momentum space. Consider two modes — one inside and one outside the horizon and attach comoving observers with both. The two observers feel a relative constant acceleration between them. Approximating the near horizon geometry by the Rindler geometry, one can easily obtain the reduced density matrix corresponding to the state in Region I of the Rindler space–time by tracing over the states in the causally disconnected Region II. The reduced density matrix reads (see for example [18])

$$\begin{aligned}\rho_I &\sim \frac{1}{\cosh^2\alpha}\sum_n \tanh^{2n}\alpha \ |n\rangle^I \langle n|^I \\ &= (1-e^{-2\pi\omega/H})\sum_n (e^{-2\pi\omega/H})^n \, |n\rangle^I \langle n|^I \end{aligned} \tag{12}$$

The above is a thermal state with Unruh temperature $T_{Unruh} = H/(2\pi) = T_{dS}$. This is correct up to terms $\mathcal{O}(\alpha = \tanh^{-1}[e^{-\pi k/(aH)}])$ (reason for '$\sim$' in the first line of the above equation) which are rather small for $k \geq aH$, where ω has been traded off in favour of k and now expressed as physical momentum. The state of the field in the infinite past is assumed to be the vacuum state which corresponds to the two mode squeezed state (as in quantum optics) in the late future. We note that approximating the near horizon geometry to be Rindler geometry is an approximation only for modes in the neighbourhood of k_* i.e. at the time of horizon exit. This would form a small set of modes but this is precisely the set that one is usually interested in the context of inflationary cosmology. When employing general initial conditions, the exact relation between quantities like scale factor $a(t)$, momentum k and Hubble constant H may change, depending upon the exact nature and profile of the adopted or assumed initial state. However, the above relation is still expected to hold modulo small corrections which as eluded to above are neglected. A more careful analysis would warrant the use of exact relations and may in principle help differentiate between various initial states.

A squeezed state can be constructed by acting the coherent state displacement operator $D(\alpha)$ followed by the squeezing operator $S(\xi)$ on the vacuum state $|E\rangle$. Since for the harmonic potential the coherent state stays a coherent state under time evolution (only the complex parameter changes by a phase), it is not difficult to establish that a squeezed state also remains a squeezed state with the only change that the parameters α and ξ pick up phase factors with phases ωt and $2\omega t$ respectively. This conclusion is expected to change when non-trivial interactions are considered. The detailed investigation of effect of interactions is beyond the scope of this note and we leave it to a separate study [24]. For the time being we consider a theory with two fields with some reasonable interactions, both self interactions and with each other. In this toy theory, the future squeezed state is now going to be rather complicated and will depend on the creation and annihilation operators of the two fields. On generic grounds there is no reason for the two point functions of these two fields evaluated in this complicated squeezed state to be identical. It should therefore be not surprising that the power spectra of the two fields have different momentum dependence. In particular, it may be possible, at least in a toy theory with tuned interactions, that power spectrum of one field is nearly scale invariant while the other field has a power spectrum with a positive scale dependence.

All what has been discussed above can now be applied to the case of inflation. In the single field inflation, there are two fields: the inflaton field, a scalar with a potential which in principle can have self-interactions, and the graviton field, a symmetric transverse traceless tensor. In the language of the toy theory considered above, the inflaton field is the one whose power spectrum is nearly scale invariant ($n_S - 1 = -2\epsilon$) while the graviton has a blue tilted power spectrum with $n_T \sim 1$ and r consistent with the PLANCK limit.[5] This scenario has been shown to be consistent with the cosmological data, including the BICEP2 results when the *in* vacuum is chosen to be the Euclidean vacuum and the *out* vacuum is chosen to be the one corresponding to a static observer (see S. Mohanty and A. Nautiyal in [8]). This corresponds to the identification $\alpha' = -\pi k/(aH)$. As we have discussed above, instead of interpreting as a vacuum state, it is more appropriate to view the transformed state as a squeezed state. Another advantage in considering these states as excited states rather than alternate vacuum states is that by definition a vacuum state would have to be annihilated by the annihilation operators of both the fields in the toy theory. Therefore one would have to invoke some mechanism by which the power spectrum of only one of them gets significant corrections. As argued above, the squeezed state interpretation has the potential to naturally address this aspect.

One therefore notes that the discussion and the construction outlined above provides strong theoretical basis to scenarios where correlation functions are evaluated in more general states. The

[4] Strictly speaking, for $\alpha_n = \alpha$ i.e. independent of n, the overlap between the vacuum and the α state is zero. This orthogonality would therefore forbid their interpretation as excited states. However, in an effective theory sense, one can still have this description allowed and this is the standpoint what we adopt here.

[5] Whether it is possible to have such a scenario realised with a realistic inflaton potential remains to be established. For the present purpose we assume that it does realise.

states, though, cannot be chosen completely arbitrarily and are supposed to be constructed following a well defined set of rules. General mixed states have been considered on a somewhat phenomenological level. They should be subjected to the checks dictated by the prescribed set of rules. It is still possible to have multiple such states which turn out to be consistent with the data at present. As an example, consider a different scenario consistent with data where the scalar spectrum also has a non-negligible momentum dependence together with the tensor spectrum. As we have argued above, it may be possible to achieve such a scenario by appropriately choosing the potential. The question to be addressed next is which one of the two describes our universe i.e. which set of initial conditions best describes the physical universe that we observe. The answer, and the capability to differentiate, lies in the higher point correlation functions and therefore future measurements or limits on the non-Gaussianities will be essential in settling this issue. At this point it seems worthwhile to point out that there have been some attempts to exploit higher correlation functions in order to distinguish between the standard Bunch–Davies and non-Bunch–Davies states and it is observed that the two cannot be easily differentiated (see e.g. [19]). This is somewhat different from what is stated above as there it is trying to differentiate two sets of initial conditions. However, one must acknowledge that this may not be an easy task either and may require lot more precise data.

In this note we have focused our attention on the α vacuum states and their interpretation in the light of the recent results on the primordial gravitational wave observations by BICEP2 Collaboration. Non-Bunch–Davies/Euclidean vacuum states have been proposed as a solution to the observed blue tilt of the tensor power spectrum and further to reconcile the positive large value of r as announced by the BICEP2 and the PLANCK upper limit on r. From the point of view of quantum field theory, α vacua lead to various pathologies like undesirable singularity structure of the correlation functions and departure from the standard thermal behaviour of an Unruh detector. Interpreting these as excited states is an interesting possibility and we have argued that such an interpretation seems to be quite natural. It naturally leads to mixed states and this is consistent with the presence of the de Sitter horizon. We have further pointed out that it may be possible to construct potentials and interactions, albeit fine tuned, which can lead to desired features in the scalar and tensor power spectra. Whether in practice it is easy to realise such a construction remains to be explored. It is worth mentioning that on the theoretical side, there have been recent studies on computing entanglement entropy in de Sitter space both in the Euclidean [20] and α vacua [21] (see also [22]). It is found that the entanglement entropy increases monotonically with the value of α. This therefore, at least as a matter of principle, provides a handle on α. It has been argued that the change in entropy as seen by two observers in two patches of the de Sitter space is the same [23]. The two observations above can be used to rule out certain non-Bunch–Davies states. The basic reasoning would be as follows: start with a generic non-standard initial state and compute the entanglement entropy. This should then be compared to the standard case of entanglement entropy computed with Bunch–Davies state. As pointed out above, a genuine α excited state would have larger entanglement entropy, and this would provide a direct handle on α. An ad hoc non-standard initial state, in general, is not going to conform to this expectation. Coupled with this, the next test would be to compute the change in entropy in the two patches (say northern and southern diamonds of de Sitter space). The expectation is that the change in entropy should be the same. Also, one could compute the change in energy in the two diamonds when, say, one mode is excited in the northern diamond. The difference between the change in energies in the two patches should simply be the single mode excitation energy. All these expectations, in general, will not be borne out by an arbitrary initial state, considered as a phenomenological starting point for explaining certain features of inflationary picture. We thus have some kind of a theoretical consistency expected of initial states which can be used to rule out certain variety of initial states. It would thus be interesting to consider an interacting theory and compute the entanglement entropy in the squeezed states, and also the change in entropy. Such a theoretical exercise, considering these two tools together, may perhaps shed some further light on the nature of these states in such a context. A particularly relevant quantity would be the entanglement entropy in the momentum space and its scaling properties. This will be investigated elsewhere [24].

References

[1] G. Hinshaw, et al., WMAP Collaboration, Astrophys. J. Suppl. 208 (2013) 19, arXiv:1212.5226 [astro-ph.CO].

[2] P.A.R. Ade, et al., Planck Collaboration, arXiv:1303.5082 [astro-ph.CO].

[3] J.C. Mather, E.S. Cheng, D.A. Cottingham, R.E. Eplee, D.J. Fixsen, T. Hewagama, R.B. Isaacman, K.A. Jesnsen, et al., Astrophys. J. 420 (1994) 439;
C.L. Bennett, A. Banday, K.M. Gorski, G. Hinshaw, P. Jackson, P. Keegstra, A. Kogut, G.F. Smoot, et al., Astrophys. J. 464 (1996) L1, arXiv:astro-ph/9601067.

[4] P.A.R. Ade, et al., BICEP2 Collaboration, arXiv:1403.3985 [astro-ph.CO].

[5] A.H. Guth, Phys. Rev. D 23 (1981) 347;
A.D. Linde, Phys. Lett. B 108 (1982) 389.

[6] D. Baumann, arXiv:0907.5424 [hep-th].

[7] C. Cheng, Q.-G. Huang, arXiv:1403.5463 [astro-ph.CO];
M. Gerbino, A. Marchini, L. Pagano, L. Salvati, E. Di Valentino, A. Melchiorri, arXiv:1403.5732 [astro-ph.CO];
K.N. Abazajian, G. Aslanyan, R. Easther, L.C. Price, arXiv:1403.5922 [astro-ph.CO];
C. Cheng, Q.-G. Huang, arXiv:1403.7173 [astro-ph.CO];
H. Li, J.-Q. Xia, X. Zhang, arXiv:1404.0238 [astro-ph.CO];
C. Cheng, Q.-G. Huang, arXiv:1404.1230 [astro-ph.CO].

[8] H. Collins, R. Holman, T. Vardanyan, arXiv:1403.4592 [hep-th];
A. Ashoorioon, K. Dimopoulos, M.M. Sheikh-Jabbari, G. Shiu, arXiv:1403.6099 [hep-th];
A. Aravind, D. Lorshbough, S. Paban, arXiv:1403.6216 [astro-ph.CO];
S. Mohanty, A. Nautiyal, arXiv:1404.2222 [hep-ph];
S. Mukohyama, R. Namba, M. Peloso, G. Shiu, arXiv:1405.0346 [astro-ph.CO].

[9] L. Hui, W.H. Kinney, Phys. Rev. D 65 (2002) 103507, arXiv:astro-ph/0109107;
J. Martin, R. Brandenberger, Phys. Rev. D 68 (2003) 063513, arXiv:hep-th/0305161;
A. Ashoorioon, K. Dimopoulos, M.M. Sheikh-Jabbari, G. Shiu, JCAP 1402 (2014) 025, arXiv:1306.4914 [hep-th].

[10] R. Bousso, A. Maloney, A. Strominger, Phys. Rev. D 65 (2002) 104039, arXiv:hep-th/0112218;
M.B. Einhorn, F. Larsen, Phys. Rev. D 67 (2003) 024001, arXiv:hep-th/0209159;
M.B. Einhorn, F. Larsen, Phys. Rev. D 68 (2003) 064002, arXiv:hep-th/0305056;
T. Banks, L. Mannelli, Phys. Rev. D 67 (2003) 065009, arXiv:hep-th/0209113;
U.H. Danielsson, JHEP 0212 (2002) 025, arXiv:hep-th/0210058;
J. de Boer, V. Jejjala, D. Minic, Phys. Rev. D 71 (2005) 044013, arXiv:hep-th/0406217;
M. Spradlin, A. Strominger, A. Volovich, arXiv:hep-th/0110007.

[11] T.S. Bunch, P.C.W. Davies, Proc. R. Soc. Lond. A 360 (1978) 117.

[12] E. Mottola, Phys. Rev. D 31 (1985) 754;
B. Allen, Phys. Rev. D 32 (1985) 3136.

[13] G.W. Gibbons, S.W. Hawking, Phys. Rev. D 15 (1977) 2738.

[14] W.G. Unruh, Phys. Rev. D 14 (1976) 870.

[15] See for example M.O. Scully, M.S. Zubairy, Quantum Optics, Cambridge University Press, Cambridge, 1997;
S.M. Barnett, P.M. Radmore, Methods in Theoretical Quantum Optics, Oxford University Press, New York, 1997;
D.F. Walls, G.J. Milburn, Quantum Optics, Springer-Verlag, Berlin, 1994;
C.C. Gerry, P.L. Knight, Introductory Quantum Optics, Cambridge University Press, Cambridge, 2008.

[16] A. Albrecht, P. Ferreira, M. Joyce, T. Prokopec, Phys. Rev. D 50 (1994) 4807, arXiv:astro-ph/9303001.

[17] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47 (1935) 777.

[18] W.G. Unruh, R.M. Wald, Phys. Rev. D 29 (1984) 1047;
L.C.B. Crispino, A. Higuchi, G.E.A. Matsas, Rev. Mod. Phys. 80 (2008) 787, arXiv:0710.5373 [gr-qc].
[19] R. Flauger, D. Green, R.A. Porto, JCAP 1308 (2013) 032, arXiv:1303.1430 [hep-th];
A. Aravind, D. Lorshbough, S. Paban, JHEP 1307 (2013) 076, arXiv:1303.1440 [hep-th];
K. Sugimura, E. Komatsu, JCAP 1311 (2013) 065, arXiv:1309.1579 [astro-ph.CO].
[20] J. Maldacena, G.L. Pimentel, JHEP 1302 (2013) 038, arXiv:1210.7244 [hep-th].
[21] S. Kanno, J. Murugan, J.P. Shock, J. Soda, arXiv:1404.6815 [hep-th];
N. Iizuka, T. Noumi, N. Ogawa, arXiv:1404.7487 [hep-th].
[22] W. Fischler, S. Kundu, J.F. Pedraza, arXiv:1311.5519 [hep-th].
[23] M.B. Einhorn, M. Mahato, Phys. Rev. D 73 (2006) 104035, arXiv:gr-qc/0506020.
[24] N. Mahajan, in preparation.